Approximation and Online Algorithms with Applications.

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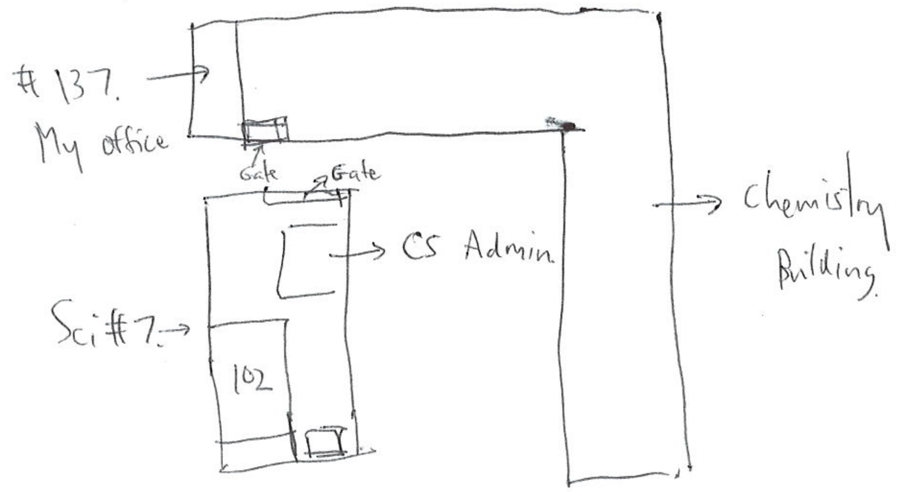
4810-1183 Approximation and Online Algorithms with Applications.

Vorapong Suppakitpaisarn. Chemistry Building #137.



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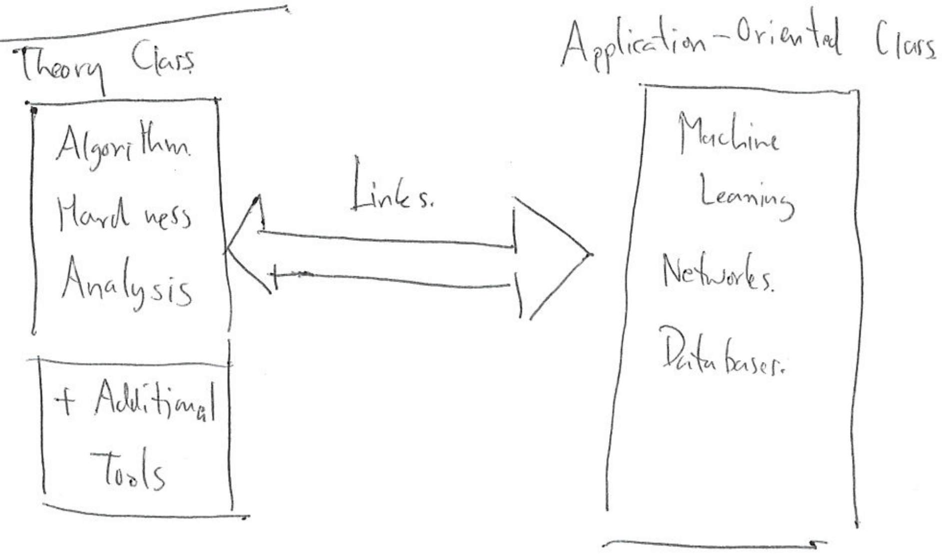
[Today after this class "Ryugaku no susume" event at Eng #1, Room #15]

<u>What do we aim at this class?</u>

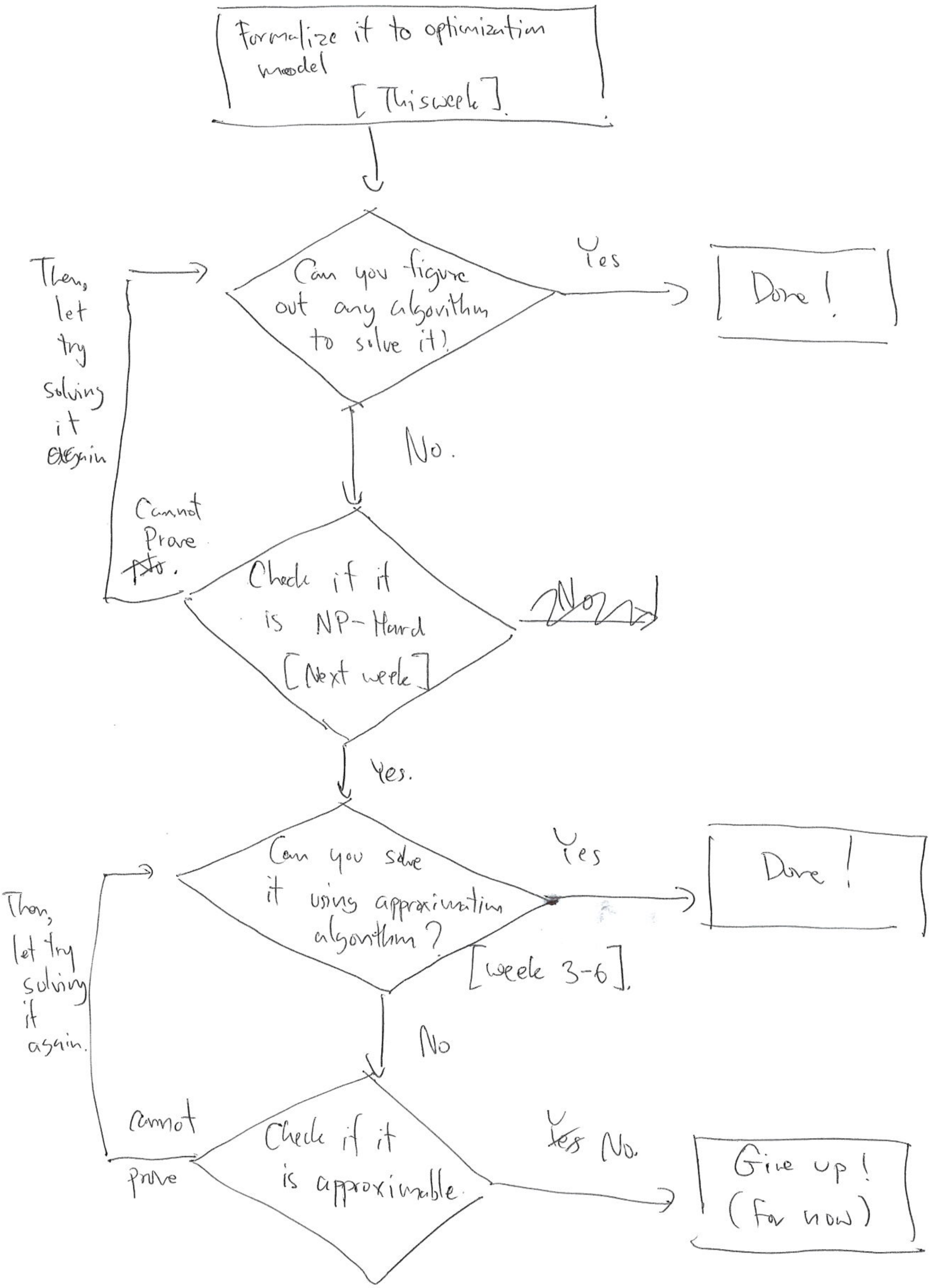


Schedule

4/11	Course Introduction, Optimization Models.
4/18.	NP-Hardness.
4/25	Approximation Algorithms. and PTAS.
5/2	" ——— "
5/9	" ——— "
5/16	No Class
5/23.	Approximation Algorithms and PTAS.
5/30.	Quiz on Approximation Algorithms. [50% of grade]
6/6.	Inapproximability.
6/13	Online Algorithms.
6/20	Online Algorithms.
6/27.	Online Algorithms.
7/4	Research Topics.
7/11	Final Examination on. Online Algorithms. [50% of grade].
7/18.	Discussion

Please e-mail me if you cannot attend the quiz and final exam on 5/30 and 7/11.

How to solve your IST problem?



Relationship with other courses.

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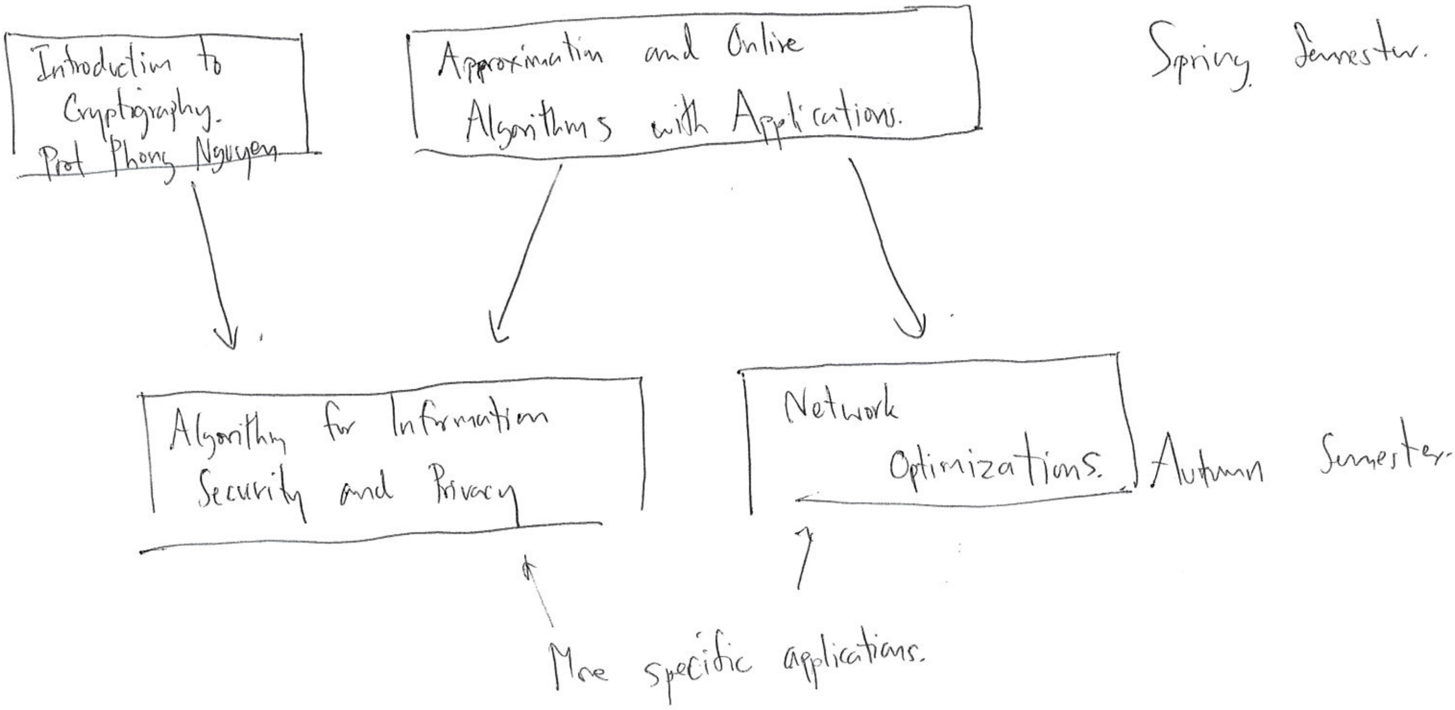


Online algorithm

(1)

$$S_1, \dots, S_i \longrightarrow \boxed{\text{Online Algorithm}} \longrightarrow d_1, \dots, d_i.$$

Input: $S_1, \dots, S_n$ where S_i is a resume of secretary i

Output: $d_1, \dots, d_n \in \{\text{take}, \text{not-take}\}$.

Optimization Models

Problem in Practical Research

What is given? [Input]. Math formulas.

What do we want? [Output] Math formula

What can the output be? [constraints]. Math formula

What do we want to maximize / minimize? [Objective Function] Math Formula

Algorithm Development

solutions.

Example

Problems in Practical Research

- We can eat only Gyu-don or Melon-pan.
- Gyu-don and Melon-pan contains different amount of carbohydrates and proteins.

Input:

- Amount of carbohydrates and proteins in one unit of Gyu-don and Melon-pan.

  $a_{G,C}, a_{G,P} \in \mathbb{R}_{\geq 0}$

  $a_{M,C}, a_{M,P} \in \mathbb{R}_{\geq 0}$.

- Cost of Gyu-don and Melon-pan per unit.

  $c_G, c_M \in \mathbb{R}_{\geq 0}$

- Amount of Carbohydrates and Proteins needed per day.

  $b_C, b_P \in \mathbb{R}_{\geq 0}$.

Output:

- Amount of Melon-pan and Gyu-don that should be taken in one day.

  $x_G, x_M \in \mathbb{R}_{\geq 0}$.

Objective function:

- Minimize Cost in each day.

Minimize $c_G x_G + c_M x_M$.

cost for x_G Gyu-don

cost for x_M Melon-pan

Constraints.

- We have to take protein moe than the required amount.

$$\underbrace{a_{G,P} \cdot x_G}_{\text{Protein from } x_G \text{ Gyo-don}} + \underbrace{a_{M,P} \cdot x_M}_{\text{Protein from } x_M \text{ Melon-Pan}} \geq b_P.$$

$$\underbrace{a_{G,C}\, x_G}_{\text{Carbohidate from } x_G \text{ Gyo-don}} + \underbrace{a_{M,C} \cdot x_M}_{\text{Carbohidates from } x_M \text{ Melon-Pan}} \geq b_C.$$

~~Protein from x_G~~

Consider only the math formula.

Input.

$$A = \begin{bmatrix} a_{G,C} & a_{M,C} \\ a_{G,P} & a_{M,P} \end{bmatrix}$$

$$b = \begin{bmatrix} b_C \\ b_P \end{bmatrix} \qquad c = \begin{bmatrix} c_G \\ c_M \end{bmatrix}$$

Output

$$x = \begin{bmatrix} x_G \\ c_M \end{bmatrix}$$

Objective Function.

$$c_G \cdot x_G + c_M \cdot x_M = c^T \cdot x.$$

Constraints.

$$a_{G,P} \cdot x_G + a_{M,P} \cdot x_M \geq b_P$$

$$a_{G,C}\, x_G + a_{M,C}\, x_M \geq b_C$$

$$= \begin{bmatrix} a_{G,P} \cdot x_G + a_{M,P} \cdot x_M \\ a_{G,C} \cdot x_G + a_{M,C} \cdot x_M \end{bmatrix} \geq \begin{bmatrix} b_P \\ b_C \end{bmatrix}$$

$$\begin{bmatrix} a_{G,P} & a_{M,P} \\ a_{G,C} & a_{M,C} \end{bmatrix} \begin{bmatrix} x_G \\ x_M \end{bmatrix} \geq \begin{bmatrix} b_P \\ b_C \end{bmatrix}$$

$$A \cdot x \geq b.$$

Generalize the problem.

<u>Input</u> ~~A, b, c~~ Matrix A, Vectors b, c.

<u>Output</u> Vector x

<u>Objective Function</u> $c^T \cdot x$.

<u>Constraints</u> $A \cdot x \geq b$.

Linear Programming.

↓

Can be solved efficient by Simplex.

or interior-point method.

<u>Library</u>: CPLEX (IBM) or MLCP.

<u>Example</u> <u>Input</u>: $A = \begin{bmatrix} a_{G,C} & a_{M,C} \\ a_{G,P} & a_{M,P} \end{bmatrix} = \begin{bmatrix} 300 & 400 \\ 200 & 50 \end{bmatrix}$

$$b = \begin{bmatrix} b_C \\ b_P \end{bmatrix} = \begin{bmatrix} 3000 \\ 1000 \end{bmatrix}$$

$$c = \begin{bmatrix} c_G \\ c_M \end{bmatrix} = \begin{bmatrix} 300 \\ 150 \end{bmatrix}$$

<u>Output</u>: $x = \begin{bmatrix} x_G \\ x_M \end{bmatrix}$.

<u>objective function</u>: $c^T x = 300\, x_G + 150\, x_M$.

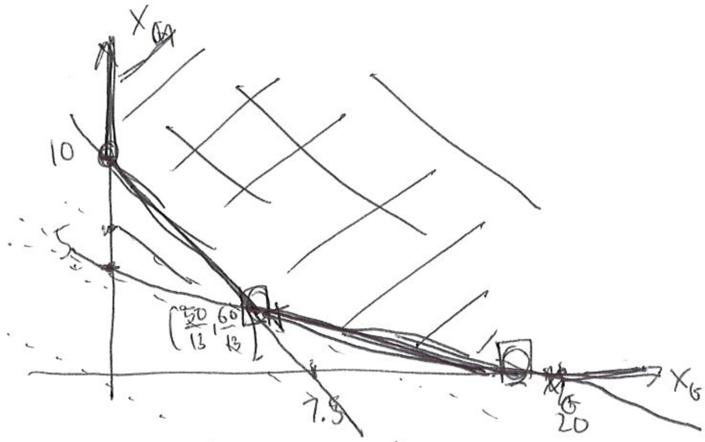
<u>Constraints</u>: $Ax \geq b$.

$$\begin{bmatrix} 300 & 400 \\ 200 & 50 \end{bmatrix} \begin{bmatrix} x_G \\ x_M \end{bmatrix} \geq \begin{bmatrix} 3000 \\ 1000 \end{bmatrix}$$

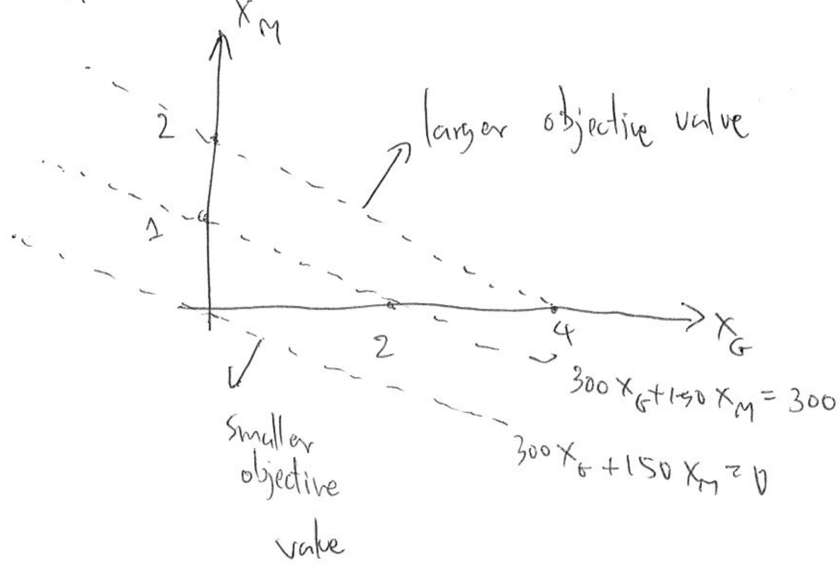
$$300\, x_G + 400 \cdot x_M \geq 3000$$

$$200 \cdot x_G + 50 \cdot x_M \geq 1000.$$

$$300\, x_G + 75\, x_M \geq 1500$$



$300x_G + 150x_M = 600$



Simplex

1: Start at some pivot of the feasible set.

2: Move to ~~the~~ a neighbor pivot which has a smaller objective value

3: If no neighbor with smaller value, we are done.

$$\text{Best } x = \begin{bmatrix} 50/13 \\ 60/13 \end{bmatrix} = \begin{bmatrix} x_G \\ x_M \end{bmatrix}.$$

We should have 50/13 Gyu-don and 60/13 Melon-pan.